Int. J. Heat Mass Transfer. Vol. 11, pp. 1753–1760. Pergamon Press 1968. Printed in Great Britain

MASS TRANSFER IN THE CASE OF INTERFACIAL TURBULENCE INDUCED BY THE MARANGONI EFFECT

E. RUCKENSTEIN
Polytechnical Institute, Bucharest, Rumania

(*Received* 8 *April* 1968)

Abstract—A discussion concerning the mechanism of mass transfer is made by means of a physical model. In this model one considers that owing to interfacial turbulence a roll-cell structure is organized near the interface. The elements of liquid have a translatory motion with the average local velocity and a circulatory one in the roll-cell. Two limiting cases are discussed: (1) the elements of liquid keep their individuality without any change during the motion along the inside paths of the roll-cell; (2) the elements of liquid are completely refreshed during their inside motion. In the first limiting case the rate of mass transfer is increased as compared to that predicted by the penetration theory, since the time interval which an element of liquid spends at the interface is reduced. The second limiting case is similar to the renewal model. It is shown that these limiting cases have physical existence. By using a simulation procedure one concludes that the thickness of the roll-cell is of the same order of magnitude as its length. It is stressed that the treatment may be extended to wave motion and turbulent motion.

NOMENCLATURE

c,	concentration;
c_i,	value of c at the interface;
c_0,	value of c in the bulk;
C,	$= (c - c_0)/(c_i - c_0)$
D,	diffusion coefficient;
F,	$= \{[\partial(u/U)]/\partial\mu\}_{\mu=0}$;
k,	average mass-transfer coefficient;
N,	mass flux at the interface;
t,	time of contact between phases;
u,	x component of velocity;
u_0,	average value of u_i over two successive paths;
u_i,	value of u at the interface;
U_i,	value of u at the interface without the Marangoni effect;
U', U'',	average values of u_i over one of two successive paths;
U,	$= U'$ or U'';
v,	y component of velocity;
x,	distance along the interface having the origin at the beginning of each path;
x_0,	the length of a path along the interface;
y,	distance at the interface.

Greek symbols

α,	arbitrary numerical constant selected equal to 2;
δ,	thickness of diffusion boundary layer;
η,	y/δ;
μ,	$y\sqrt{U/\nu x}$;
ν,	kinematic viscosity;
ρ,	density;
σ,	interfacial tension;
$\Delta\sigma$,	difference between the value of σ at the end and at the beginning of each path. When the refreshing is complete $\Delta\sigma$ is equal to the difference between the interfacial tension at the equilibrium concentrations from the interface and the

interfacial tension between the liquid phases at their bulk concentrations;

τ, shear stress at the interface.

Subscript

j, for the two phases ($j = 1, j = 2$).

INTRODUCTION

THE MARANGONI effect is a consequence of the tendency of an interface to relapse into a state where the free energy is a minimum through expansion of the regions of a low interfacial tension and the contraction of those with a high one. The surface tension gradient leads to a shear stress which affects the motions in the vicinity of the interface and therefore the mass transfer between the two phases.

In the laminar case the velocity generated by the Marangoni effect is of importance as compared to the velocity of the interface only for sufficiently small values of the Reynolds number; this is the conclusion of several theoretical papers, concerning, either the effect of surface active agents on mass transfer [1, 2], or the effect of a surface tension gradient (caused by the mass transfer between phases) in rectification or extraction [3]. The directional dependence of the mass-transfer coefficient obtained in [3] as a consequence of the Marangoni effect, should be noted.

The influence of the Marangoni effect on mass transfer is particularly important when it induces a spontaneous agitation known under the name of interfacial turbulence in the vicinity of the interface. The occurrence of turbulence at the interface between two phases was explained by Sternling and Scriven [4] who have proved that it is a consequence of hydrodynamic instability. They have succeeded to determine in a particular case (non-flow systems) the conditions under which small perturbations, always present in any system, are amplified or damped by the Marangoni effect. Any perturbation generates convection currents which will cause concentration changes at the interface and, consequently, local changes of the interfacial tensions. Depending on the manner in which the interfacial tension depends on concentration, the movements, generated by the Marangoni effect may amplify those caused by the perturbation—and in that case effects of a spontaneous agitation will appear in the vicinity of the interface—or they may damp those caused by the perturbation and in that case no spontaneous agitation will occur. Sternling and Scriven's analysis shows, among others, that some systems may be stable with solute transfer in one direction and instable with the transfer in the opposite direction and consequently it suggests that there exists a directional dependence of the mass-transfer coefficient. Such a directional dependence was observed experimentally by Grassman and Anderes by introducing into a boiling binary liquid vapour bubbles of one or another component [5]; also by Olander and Reddy [6] who have studied the nitric acid transfer from an aqueous solvent to an organic one and from an organic solvent to an aqueous one in a stirred vessel operated in a steady state. In the cases in which interfacial turbulence occurs, the mass-transfer coefficient depends on the driving force between the two phases. A dependence of the mass-transfer coefficient on the driving force was also obtained by Sawistowski and Goltz [7] who have studied the quasi-steady transfer of a solute (acetic acid, etc.) from individual drops of benzene, quasi-continuously formed and withdrawn, to an aqueous phase moving with a small velocity; by Maroudas and Sawistowski [8] who have investigated the separate and simultaneous transfer of phenol and propionic acid between two laminar co-current streams of carbon tetrachloride and water, and by Bakker *et al.* [9] who have investigated the transfer of acetic acid or acetone between two laminar co-current streams of water and an organic phase, or between carbon tetrachloride and water.

The instability theory is able to explain the occurrence of interfacial turbulence; it also permits the explanation, from a qualitative point of view, of some unexpected features as is for instance the directional dependence of the mass-transfer coefficient, but it is unable to provide information concerning the velocity distribution in the state which has physical existence (the stable state) and, therefore, concerning the corresponding rate of mass transfer. A similar difficulty exists for the usual turbulent flow (bulk turbulence). Stimulated by the treatments based on physical models, used for the analysis of the rate of mass transfer in the case of bulk turbulence [10–12], we shall examine, by using similar means, the problem of the rate of mass transfer when interfacial turbulence is occurring. Though this analysis does not lead to an equation able to predict the mass-transfer coefficient, it throws, however, some light on the mechanism of the mass transfer. It may be also stressed that the present treatment can be extended also to other cases as are for instance the case of wave motion or the turbulent case (bulk turbulence).

THE PHYSICAL MODEL

Let us consider two phases flowing in co-current. The velocity fluctuations due to interfacial turbulence bring elements of liquid from the bulk in some regions of the interface. In this manner zones of different interfacial tension appear and as a consequence also motions directed from regions of low to those of larger interfacial tension. Each element of liquid which reaches the interface generates, in the two directions, motions of opposite sign. Since we have a large number of such elements of liquid distributed more or less uniformly over the interface a roll cell structure superposed over the average motion of the liquid is achieved. If we assume for simplicity a two dimensional motion and that the distance between two successive regions reached by elements from the bulk is constant, the motion looks as in Fig. 1. The arrows in this figure refer to the relative motion with respect to the average motion of the liquid (defined over a succesion of two paths of length x_0). Each element of liquid has therefore a translatory motion with the local average velocity of the liquid and a circulatory one in the roll cell. It may be noted that the simplest structures observed photographically by means of a Schlieren technique have also the cellular feature from Fig. 1 [13].

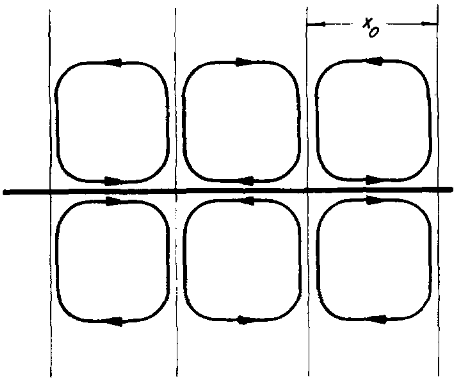

FIG. 1. The roll-cell model.

Let us consider an element of liquid which just reaches the interface; it covers along the interface a path of length x_0 and then leaves the interface and is moving along the inside paths of the roll cell. During the time it spends at the interface a mass exchange with a corresponding element of liquid from the other phase takes place, while during the inside motion a mass exchange with the bulk of liquid of the same phase is occurring.

If interfacial turbulence does not occur the average mass-transfer coefficient (the average being defined over the time of contact between phases) may be calculated by means of the well known equation [10]

$$k_j = \left(\frac{4}{\pi}\right)^{\frac{1}{2}} \left(\frac{D_j}{t}\right)^{\frac{1}{2}}. \qquad (1)$$

Two effects determine the increase of the mass-transfer coefficient when interfacial turbulence occurs. The first must be linked to the circulatory motion since it reduces the time which an element of liquid spends at the interface. If the circulation time is denoted by θ and the time the element of liquid covers the distance x_0 along the interface by θ', the ratio

θ'/θ represents the fraction of time which an element of liquid spends at the interface and exchange mass with the other phase. The second effect must be linked to the refreshing of the element of liquid during the lapse of time it is moving along the inside paths of the roll cell.

Two limiting cases can be treated in a very simple manner: (1) the elements of liquid keep their identity without any change during the motion along the inside paths of the roll cell*; (2) the elements of liquid are completely refreshed during their inside motion and consequently they have the concentration of the bulk when they reach the interface.

In the first limiting case only the time of contact of the considered element of liquid with the other phase must be changed in equation (1), which becomes

$$k_j = \left(\frac{4}{\pi}\right)^{\frac{1}{2}} \left(\frac{D_j}{t(\theta'/\theta)}\right)^{\frac{1}{2}} \tag{2}$$

In the second limiting case the elements of liquid which reach the interface are continuously fresh ones and the situation becomes similar to that encountered in the renewal models. Over each path is valid in this case an equation of the form (1). The time t must be, however, replaced by the time interval θ' (i.e. the time interval from the circulation time which the element covers along the interface the distance x_0). If u_0 is the average velocity of the interface (defined over a succession of two paths) and u_i the local velocity

$$\theta' = \int_0^{x_0} \frac{\mathrm{d}x}{|u_0 - u_i|}. \tag{3}$$

For the mass-transfer coefficient one may write the equation

$$k_j = \left(\frac{4}{\pi}\right)^{\frac{1}{2}} \left(\frac{D}{\theta'}\right)^{\frac{1}{2}} \tag{4}$$

As shown in Appendix A, equation (4) is valid in the present case if one replaces θ' by $x_0^2/\int_0^{x_0} |u_0 - u_i|\,\mathrm{d}x$. The results obtained by both this last expression and equation (3) are however about the same.

It may be noted that in the first limiting case (equation (2)) the mass transfer coefficient depends on the time of contact between the two phases, while in the second limiting case it is independent on this time. It is to be expected that for small values of t (owing to the mass exchange between the elements of liquid and their own phase) the elements which reach the interface are fresh ones and therefore equation (4) is valid. As the time t increases, the inside regions with which the considered elements of liquid exchange mass become more and more saturated, since the convective mass transfer into these regions is larger than the diffusional transfer from them. For this reason the refreshing of the elements of liquid becomes increasingly weaker and the conditions of validity of the first extreme case, equation (2), are increasingly achieved.† Simultaneously, owing to the weak refreshing the influence of the Marangoni effect becomes weaker and the interfacial turbulence decay.

For very large values of t, the diffusion in the region beyond the roll cell must determine the rate of mass transfer. Such a situation is, however, not encountered in the usual cases.

Above it was assumed that the penetration of interfacial turbulence is restricted to the thickness of one roll cell. It is, however, possible (and some photographs obtained by a Schlieren technique confirm this fact [7, 8]) that the interfacial turbulence should penetrate very deep into the liquid. The model used may be extended to this case too, by considering, however, that mass is transferred from the regions near the inside path of the first roll cell

---

* A weak change is, however, necessary, otherwise it is not possible to entertain a gradient of concentration along the interface in the vicinity of the points the element of liquid reach the interface and therefore to entertain the interfacial turbulence.

† The effect of uniformization of concentration in the considered element of liquid during the time interval θ–θ' is here not taken into account, though it may be of importance.

to more distant regions from the interface by a convective motion (possibly by a second roll cell), from these new regions to more distant ones, and so on. In this case the refreshing of the considered element of liquid during its motion along the inside paths of the first cell is practically complete and one may use for the mass-transfer coefficient equation (4).

DISCUSSION

Though the real mechanism of mass transfer is probably intermediate, there exist in the literature some experiments which appear to be compatible with one or another of the above-mentioned limiting cases. Experiments of Bakker *et al.* [9] seem to be in agreement with the first limiting case, while those of Sawistowski and Goltz [7] with the second limiting case. In the first experiment the number of rotations which undergo an element of liquid is probably large (the mass transfer is studied for two co-current liquids with a sufficiently long contact surface). In the second the authors' photographs show a deep penetration of interfacial turbulence.

The experiments of Bakker *et al.* [9] lead to an equation having the form of equation (2). In order that a more quantitative comparison be made it is necessary to evaluate the time interval θ' and θ. For the time being it is difficult to achieve theoretically this aim and, since the real structure is a very complex one, also experimentally. Nevertheless, some information was obtained by way of simulating the simplified physical model. The simulation was achieved by generating artifically the Marangoni effect by means of a procedure proposed in [14]. The method consists in the continuous feeding through a capillary of a liquid having a lower surface tension over the surface of another stagnant one (or over the interface of two liquids) into which it dissolves. If several sources of Marangoni effect of the above type are used it is possible to simulate the present physical model since a roll cell structure, as sketched in Fig. 1, is organized. In Fig. 2 one of the photographs obtained in this laboratory with a Schlieren technique by Suciu and Smighelschi is given. The upper layer is paraffin oil and the lower layer is ethylene glycol. At interface ethyl acetate is fed continuously through two glass capillaries. From photographs of the above type one may conclude that the thickness of the roll-cell is approximately equal to x_0. If for a first evaluation one assumes that the circulation velocity is the same along the contour of the roll-cell, the value $\frac{1}{4}$ is obtained as an order of magnitude for θ'/θ. It may be noted that in this manner there results for the ratio between k_j given by equation (2) and k_j given by the usual penetration theory the value 2 which is of the order of magnitude obtained experimentally by Bakker *et al.* [9]. The increase (observed experimentally) of the mass-transfer coefficient with the driving force is due to a larger circulation velocity and therefore to a larger refreshing.

In the experiments carried out by Sawistowski and Goltz it seems that the velocity generated by the Marangoni effect must prevail. A very approximate calculation (see Appendix B) enables one to evaluate this velocity as a function of $\Delta\sigma$, x_0, ν_1, ν_2, ρ_1 and ρ_2. One obtains

$$0{\cdot}88(\rho_1\nu_1^{\frac{1}{2}} + \rho_2\nu_2^{\frac{1}{2}})\,x_0^{\frac{1}{2}}|U|^{\frac{3}{2}} = |\Delta\sigma| \qquad (5)$$

For reasons mentioned above it is to be expected that in this case the elements of liquid are completely refreshed. It is therefore possible to use for k_j equation (4). Some numerical calculations of the mass-transfer coefficient are given in Table 1. They were carried out for several values of x_0. For the physical constants the values $\rho_1 = \rho_2 = 1$ g/cm^3; $\nu_1 = \nu_2 = 10^{-2}$ cm^2/s; $D = 10^{-5}$ cm^2/s were taken and for $\Delta\sigma$ values in the range appearing in the experiments of Sawistowski and Goltz. The calculated values of k_j are of the order of those obtained experimentally if $x_0 = 10^{-1} \div 10^{-2}$ cm. It may be noted that the dimensions of the cells observed experimentally [13] are in the same range. Concerning the order of magnitude there exists therefore agreement between the above evaluations and experiment.

Table 1. $\nu_1 = \nu_2 = 10^{-2}$ cm²/s; $\rho_1 = \rho_2 = 1$ g/cm³; $D = 10^{-5}$ cm²/s

$\Delta\sigma$ dynes/cm	k cm/s		
	$x_0 = 10^{-1}$	$x_0 = 10^{-2}$	$x_0 = 10^{-3}$
1	3×10^{-2}	$1{\cdot}4 \times 10^{-1}$	0·64
5	5×10^{-2}	$2{\cdot}4 \times 10^{-1}$	1·1
10	$6{\cdot}4 \times 10^{-2}$	3×10^{-1}	1·4
20	8×10^{-2}	$3{\cdot}7 \times 10^{-1}$	1·7

As a general comment it may be noted that the problems treated here are characterized by the fact that diffusion is acting not only in the vicinity of the interface but, owing to circulation motions of the above type, also at some distance from the interface. A more theoretical approach than the present one is now under way. We stress also the fact that the above treatment may be extended to other cases too: the first limiting case to wave motion* while the second limiting case to turbulent motion.

REFERENCES

1. V. G. Levich, *Physiochemical Hydrodynamics*. Prentice-Hall, New Jersey (1962).
2. E. Ruckenstein, Effect of surface active agents on the mass transfer in falling liquid films, *Chem. Engng Sci.* **20**, 853–856 (1965).
3. E. Ruckenstein, Influence of the Marangoni effect on mass-transfer coefficient, *Chem. Engng Sci.* **19**, 505–506 (1964).
4. C. V. Sternling and L. E. Scriven, Interfacial turbulence: hydrodynamic instability and the Marangoni effect, *A.I.Ch.E. Jl* **5**, 514–523 (1959).
5. P. Grassman and G. Anderes, Abhängigkeit des Stoffaustausches von der Austauschrichtung, *Chemie-Ingr-Tech.* **31**, 154–155 (1959).
6. D. R. Olander and L. B. Reddy, The effect of concentration driving force on liquid–liquid mass transfer, *Chem. Engng Sci.* **19**, 67–73 (1964).
7. H. Sawistowski and G. E. Goltz, The effect of interface phenomena on mass-transfer rates in liquid–liquid extraction, *Trans. Instn Chem. Engrs* **41**, 174–181 (1963).
8. N. G. Maroudas and H. Sawistowski, Simultaneous transfer of two solutes across liquid–liquid interfaces, *Chem. Engng Sci.* **19**, 919–931 (1964).
9. C. A. P. Bakker, F. H. Fentener van Vlissingen and W. J. Beek, The influence of the driving force in liquid–liquid extraction—a stucy of mass transfer with and without interfacial turbulence under well-defined conditions, *Chem. Engng Sci.* **22**, 1349–1355 (1967).
10. R. Higbie, The rate of absorption of a pure gas into a still liquid during short periods of exposure, *Trans. Am. Instn Chem. Engrs* **31**, 365–370 (1935).
11. E. Ruckenstein, Equations for the mass- or heat-transfer coefficient in turbulent motion, *Int. J. Heat Mass Transfer* **9**, 441–451 (1966).
12. E. Ruckenstein, A note concerning turbulent exchange of heat or mass with a boundary, *Chem. Engng Sci.* **7**, 265–268 (1958).
13. A. Orell and J. W. Westwater, Spontaneous interfacial cellular convection accompanying mass transfer: ethylene glycol–acetic-acid–ethyl acetate, *A.I.Ch.E. Jl* **8**, 350–356 (1962).
14. D. G. Suciu, O. Smighelski and E. Ruckenstein, Some experiments on the Marangoni effect, *A.J.Ch.E. Jl* **13**, 1120–1125 (1967).
15. F. K. Tsou, E. M. Sparrow and R. J. Goldstein, Flow and heat transfer in the boundary layer on a continuous moving surface, *Int. J. Heat Mass Transfer* **10**, 219–235 (1967).

APPENDIX A

The convective diffusion equation has the form:

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}. \qquad \text{(A.1)}$$

The depth of penetration by diffusion being very small one may use for u its value at the interface. The continuity equation leads for v to

$$v = -y\frac{\mathrm{d}u}{\mathrm{d}x} \qquad \text{(A.2)}$$

and equation (1) becomes

$$u(x)\frac{\partial c}{\partial x} - y\frac{\mathrm{d}u}{\mathrm{d}x}\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}. \qquad \text{(A.3)}$$

Equation (3) must be solved for the boundary conditions

$$\left.\begin{aligned} c &= c_i \quad &&\text{for} \quad y = 0 \\ c &= c_0 \quad &&\text{for} \quad y \to \infty \\ c &= c_0 \quad &&\text{for} \quad x = 0. \end{aligned}\right\} \text{(A 4)}$$

Equation (A.3) and the boundary conditions are compatible with a solution of the form

---

* The two-fold increase of the mass-transfer coefficient by the wave motion may be predicted by the above considerations.

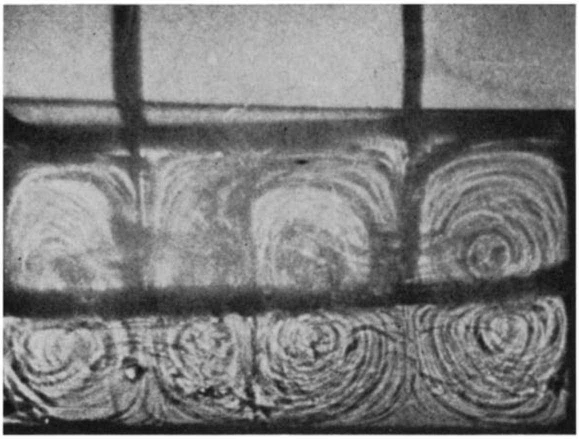

FIG. 2. Simulation of the physical model by means of two sources of Marangoni effect generation. The upper phase is paraffin oil; lower phase is ethylene glycol. Through the two capillaries (the two vertical black streaks) ethyl acetate is fed.

$$C = C(\eta) \qquad \text{where} \quad \eta = \frac{y}{\delta(x)}. \tag{A.5}$$

Indeed, the similarity variable η enables the transformation of equation (A.3) into

$$\left(\frac{u}{2}\frac{\mathrm{d}\delta^2}{\mathrm{d}x} + \delta^2\frac{\mathrm{d}u}{\mathrm{d}x}\right)\eta\frac{\mathrm{d}C}{\mathrm{d}\eta} + D\frac{\mathrm{d}^2C}{\mathrm{d}\eta^2} = 0. \tag{A.6}$$

For $C = C(\eta)$ and $\delta = \delta(x)$ we must have

$$\alpha\eta\frac{\mathrm{d}C}{\mathrm{d}\eta} + \frac{\mathrm{d}^2C}{\mathrm{d}\eta^2} = 0 \tag{A.7}$$

and

$$\frac{u}{2}\frac{\mathrm{d}\delta^2}{\mathrm{d}x} + \delta^2\frac{\mathrm{d}u}{\mathrm{d}x} - \alpha D = 0, \tag{A.8}$$

where α represents an arbitrary constant. For convenience we shall select the value 2 for this constant.

From equation (A.7) and boundary conditions (A.4) one obtains

$$C = 1 - \frac{2}{\sqrt{\pi}}\int\limits_0^\eta \mathrm{e}^{-s^2}\,\mathrm{d}s \tag{A.9}$$

and from equation (A.8)

$$\delta^2 = \frac{4D}{u^2}\int\limits_0^x u(p)\,\mathrm{d}p. \tag{A.10}$$

For the mass flux there results

$$N = -D\left(\frac{\partial c}{\partial y}\right)_{y=0} = \left(\frac{D}{\pi}\right)^{\frac{1}{2}} \times \frac{u(x)}{[\int_0^x u(p)\,\mathrm{d}p]^{\frac{1}{2}}}(c_i - c_0) \tag{A.11}$$

and for the mass-transfer coefficient, defined as an average over the path of lenght x_0,

$$k = \left(\frac{D}{\pi}\right)^{\frac{1}{2}}\frac{1}{x_0}\int\limits_0^{x_0}\frac{u(x)\,\mathrm{d}x}{[\int_0^x u(p)\,\mathrm{d}\eta]^{\frac{1}{2}}}$$
$$= \left(\frac{4D}{\pi x_0}\right)^{\frac{1}{2}}\left(\frac{1}{x_0}\int\limits_0^{x_0} u(x)\,\mathrm{d}x\right)^{\frac{1}{2}}. \tag{A.12}$$

It may be noticed that equation (A.12) has exactly the same form as that of Higbie, the average velocity replacing in this case the constant surface velocity.

APPENDIX B

The velocity at the interface when the interfacial turbulence is superposed will be evaluated by using the following two rough approximations:

(1) The velocity at the interface (though depends on x) will be replaced by its average value over the length x_0 of a path:

(2) The velocity distribution in the vicinity of the interface will be approximated for each path by that valid for a plate moving with the velocity U over a semi-infinite liquid the velocity of which is equal to U_i (the velocity of the interface without interfacial turbulence).

Owing to the Marangoni effect one must write that

$$\tau_1 - \tau_2 = -\frac{\mathrm{d}\sigma}{\mathrm{d}x} \qquad \text{for} \quad y = 0. \tag{B.1}$$

Equation (B.1) together with the equation which results from the above approximations enables one to evaluate the velocity at the interface. Since we have replaced the velocity at the interface by its average value over the path of length x_0, it is natural to replace also equation (B.1) by its average over the same path:

$$\int\limits_0^{x_0}\tau_1\,\mathrm{d}x - \int\limits_0^{x_0}\tau_2\,\mathrm{d}x = -\Delta\sigma, \tag{B.2}$$

where $\Delta\sigma$ represents the difference between the interfacial tension at the end and at the beginning of each path. When the refreshing of the elements of liquid is complete $\Delta\sigma$ is equal to the difference between the interfacial tension at the equilibrium concentration from the interface and the interfacial tension between the liquid phases at their bulk concentrations.

By approximating the velocity distributions near the boundary as mentioned above and by using for then the equations from [15] one obtains

$$2\rho_1 \nu_1^{\frac{1}{2}} U^{\frac{3}{2}} F_1 x_0^{\frac{1}{2}} + 2\rho_2 \nu_2^{\frac{1}{2}} U^{\frac{3}{2}} F_2 x_0^{\frac{1}{2}} = -\Delta\sigma \quad \text{(B.3)}$$

where

$$F \equiv F\left(\frac{U_i}{U}\right) = \left(\frac{\partial(u/U)}{\partial\mu}\right)_{\mu=0}$$

and

$$\mu = y\left(\frac{U}{\nu x}\right)^{\frac{1}{2}}$$

Some values of the function F, computed by means of the numerical solutions from [15] are given in the Table 2.

Table 2

U_i/U	0	0·62	1	1·78	∞
F	−0·44	−0·20	0	0·50	0·33

Résumé—On discute le mécanisme du transport de masse au moyen d'un modèle physique. Dans ce modèle, on considère qu'à cause de la turbulence interfaciale, une structure en rouleaux s'organise près de l'interface. Les éléments du liquide ont un mouvement de translation avec la vitesse locale moyenne et un mouvement circulatoire dans le rouleau. Deux cas limites sont discutés: (1) les éléments du liquide gardent leur individualité sans aucun changement pendant le mouvement le long des trajectoires interns du rouleau; (2) les éléments du liquide sont complètement décontaminés pendant leur mouvement interne. Dans le premier cas limite, la vitesse de transport de masse est augmentée par rapport à celle prédite par la théorie de la pénétration puisque l'intervalle de temps qu'un élément de liquide passe à l'interface est réduit. Le second cas limite est semblable au modèle du renouvellement. On montre que ces cas limites ont une existence physique. En employant un processus de simulation, on conclut que l'épaisseur du rouleau est du même ordre de grandeur que sa longueur. On insiste sur le fait que le traitement peut être étendu au mouvement ondulatoire et au mouvement turbulent.

Zusammenfassung—Auf Grund eines physikalischen Modells wird der Vorgang des Massentransports diskutiert. In diesem Modell wird angenommen, dass infolge der Grenzflächenturbulenz eine Rollzellenstruktur nahe der Grenzfläche entsteht. Die Flüssigkeitsteilchen haben translatorische Bewegung mit der örtlichen Durchschnittsgeschwindigkeit und kreisförmige Bewegung in den Rollzellen. Zwei Grenzfälle werden diskutiert: (1) Die Flüssigkeitsteilchen bewahren ihre Individualität ohne Änderung während der Bewegung auf dem Weg in der Rollzelle; (2) Die Flüssigkeitsteilchen werden während inhrer Innenbewegung vollständig erneuert. Im ersten Grenzfall nimmt der Massenstrom zu, im Vergleich zu dem nach der Durchdringungstheorie ermittelten, da die Verweildauer eines Flüssigkeitsteilchens an der Grenzfläche abnimmt. Der zweite Grenzfall ist ähnlich dem Erneuerungsmodell. Es wird gezeigt, dass diese Grenzfälle physikalische Realität besitzen. Mit Hilfe eines Ähnlichkeitsverfahrens wird festgestellt, dass die Dicke der Rollzelle von gleicher Grössenordnung ist, wie ihre Länge. Die Betrachtung kann auf Wellenbewegung und turbulente Bewegung angewendet werden.

Аннотация—С помощью физической модели рассматривается механизм массообмена. Согласно этой считают, что за счет турбулентности на границе раздела образуется структура из вращающихся ячеек вблизи поверхности раздела. Жидкость движется поступательно со средней локальной скоростью и циркуляционно во вращающейся ячейке. Рассматриваются два предельных случая: (1) элементы жидкости сохраняют свою индивидуальность, не изменяясь при движении по внутренним траекториям вращающейся ячейки; (2) элементы жидкости полностью обновляются при движении по внутренним траекториям. В первом идеальном случае скорость массообмена увеличивается по сравнению со скоростью, рассчитанной по теории проникновения, поскольку уменьшается время пребывания элемента жидкости на поверхности раздела. Второй предельный случай аналогичен обновленной модели. Показано, что эти оба предельных случая физически осуществляются. Используя метод моделирования, приходим к выводу, что толщина вращающейся ячейки имеет тот же порядок величины, что и её длина. Особенно подчеркивается тот факт, что это справедливо для волнового и турбулентного движения.